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Intelligent Vehicle-Highway Systems

Time-Variant Travel Cost Calculation Under Anticipatory Routing

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Research Activities Summary

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Abstract

A key assumption in the anticipatory routing technique proposed by Kaufman, Lee and Smith (1990) is that the travel times along links in a network is known for every epoch in the planning horizon. However, an accurate calculation or estimation of these parameters is not an easy nor obvious task, as demonstrated in a simple example problem. The relationship between travel times and routing information is inter-related, i.e., one requires routings to predict travel costs while simultaneously requiring accurate travel costs to produce optimal routings. This "tail-chasing" predicament renders the routing problem far more theoretically challenging than if travel-times are assumed to be known quantities.

An iterative solution method based on the method of successive approximations is outlined for the calculation of time-variant travel costs and the identification of a dynamic-optimal routing strategies. Strengths and weaknesses of such a fixed point approach are discussed, with special emphasis given to the identification of invariance or dynamic equilibrium in some obtained optimal routing strategy, as well as to the critical role of an accurate link capacity function in modelling complex networks. A "layering" approach similar to one proposed by Alfa (1989) is recommended for further exploration over "all or nothing" assignment in hopes of avoiding instability and infeasibility in the iterative optimization of routing strategies.

OUTLINE OF RESEARCH SUMMARY

1. THEORETICAL DIFFICULTIES IN $T_{ij}(t)$ CALCULATIONS

As shown in Kaufman, Lee and Smith (1990), dynamic-optimal routing guidance can be provided to intelligent vehicles in a network if the travel times associated with every link in the network is assumed to be data, i.e., known quantities. At some point, however, these travel times must be calculated and supplied to the model. How easily can this be done? Consider the following question:

[Q1] Starting with complete knowledge about all O-D departures and their corresponding routes over some finite planning horizon, can we calculate $T_{ij}(t)$ values?

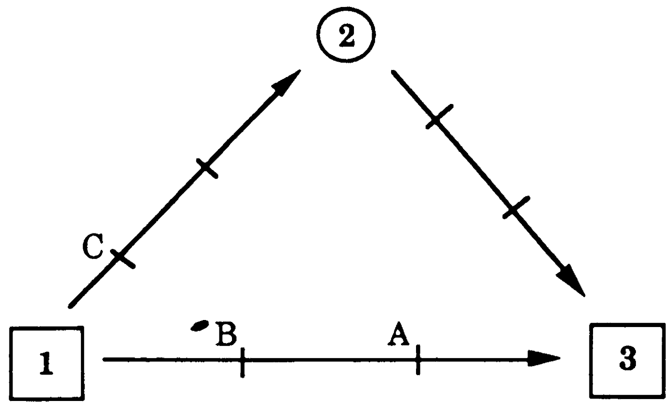
Whether or not this is possible is best illustrated by an short example problem. Consider the triangular network in Fig. 1. We assume that there is a single origin (node 1) and a single destination (node 3) for all vehicles on the network. All links are considered to be of unit length and and that these links have been divided up into thirds for bookkeeping purposes. A simple impedance function based on the standard Greenshields model is used:

$s = 4/3 - v/3$, where s = speed on the link; v = # of vehicles on the link

We know that there will be exactly six departures in a planning horizon of six epochs. At t=0, there are three vehicles already on the system, labeled A,B,C, at positions on the links indicated in Fig. 1. Also assume we know which routes the vehicles will take and construct a decision vector of six elements corresponding to the six departures in the planning horizon. Let decision "0" represent a choice to go the "long" route (node 1 - node 2 - node 3) and decision "1" represent a choice to go the "short" route (node 1 - node 3). Our decision vector is: $\mathbf{P}_1 = [\,0, 1, 0, 1, 0, 1]$.

Now we are ready to calculate $T_{ij}(t)$ values. Vehicle #1 has decision "0" so it is assigned the long route (1-2-3). Projected volumes for the first epoch on link 1-2 is now 2 vehicles (See Fig. 2). In a similar fashion, we can load vehicles #2 and #3 to the network and calculate volumes on each link (Fig 3.) We may continue in this manner to obtain volumes for the entire planning horizon.

Fig. 1 Sample Problem Formulation



- links are of unit length
- origin: 1
- destination: 3
- impedance model (Greenshields)
  speed = 4/3 - volume/3
  link capacity = 4 vehicles

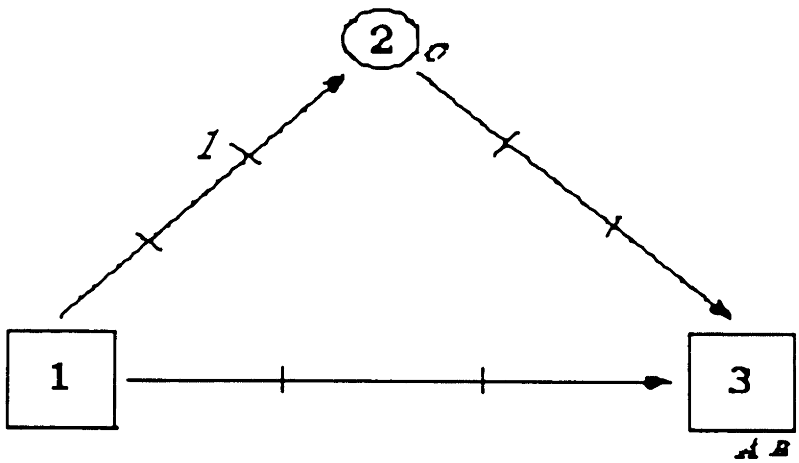
- state of system at t=0

veh#	position
A	link 1-3,pos2
B	link 1-3,pos1
C	link 1-2,pos1

- 6 departures over a planning horizon of 6 periods

veh #	departure time
1	t = 1
2,3	t = 2
4,5	t = 3
6	t = 5

Fig 2. Network status at the end of the 1st time period



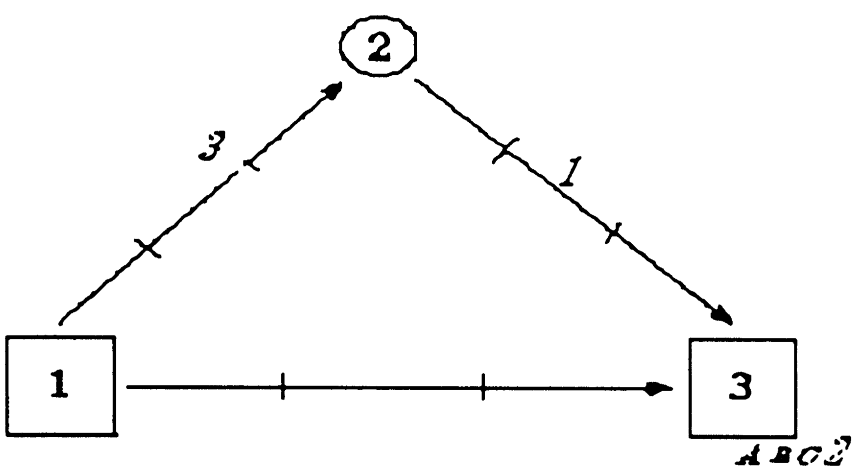
projected volumes:

link	t=0	t=1
1-2	1	2
1-3	2	2
2-3	0	0

state of system:

veh#	position
A,B	node3, finished
C	link2-3,pos0
1	link1-2,pos2

Fig 3. Network status at the end of the 2nd time period



projected volumes:

link	t=0	t=1	t=2
1-2	1	2	2
1-3	2	2	1
2-3	0	0	1

state of system:

veh#	position
A,B,C,2	node3, finished
1	link2-3,pos1.5
3	link1-2,pos2

TABLE 1. Calculated volumes under $\mathbf{P}_1$ = [0, 1, 0, 1, 0, 1].

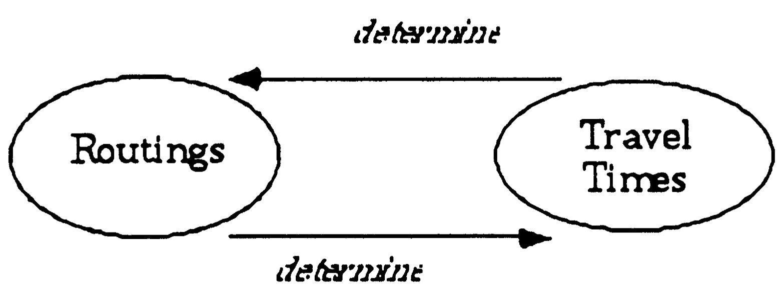
link	t=0	t=1	t=2	t=3	t=4	t=5	t=6	[no. of vehicles]
1-2	2	2	2	2	1	0	0	
1-3	1	2	1	1	0	1	0	
2-3	0	0	1	1	1	1	0	

Applying the Greenshields impedance function, we can then compute the corresponding expected travel times for each epoch in the planning horizon.

TABLE 2. Related travel times for $\mathbf{P}_1$.

link	t=1	t=2	t=3	t=4	t=5	t=6	[time units]
1-2	1.5	1.5	1.5	1	.75	.75	
1-3	1.5	1	1	.75	1	.75	
2-3	.75	.75	1	1	1	.75	

Thus, the answer to [Q1] is "yes." But how does this formulation relate to the IVHS anticipatory routing problem? Consider three separate classes of vehicles: (1) non-intelligent "background" vehicles, (2) intelligent vehicles on the road previously routed, and (3) intelligent vehicles yet to come onto the system. At any time t, calculating $T_{ij}(t)$ values based on classes (1) and (2) would be no more difficult than our sample problem. Additionally, if the vehicles in class (3) were assigned routes according to some volume independent shortest path algorithm, again this problem is not theoretically more difficult than our sample problem. However, once we introduce the concept of routing the class (3) vehicles in a volume-dependent shortest path manner, then the placement of those expected vehicles alters the $T_{ij}(t)$ values themselves and therefore the very basis for the construction of a shortest path is constantly being changed. In other words, we need routings to predict $T_{ij}(t)$ values and $T_{ii}(t)$ values to produce routings! (Fig. 4)



2. A SUCCESSIVE APPROXIMATION APPROACH

We may construct an iterative evaluation process between the determination of routing strategies and the calculation of $T_{ij}(t)$ values. That is, we can choose some initial decision vector and use it to evaluate the $T_{ij}(t)$ values. These resultant $T_{ij}(t)$ values can then be provided to the anticipatory routing algorithm to provide the next iteration's decision vector. The process continues until convergence to a least-cost routing is obtained. Here is a rough outline of such an iterative procedure:

<u>Table 3. Rough Outline of An Iterative Procedure</u>

1. Initialization
   - -compute planning horizon (H)
   - -compute number of departures in H, (n)
   - -generate initial point in the decision space, $\mathbf{P}_1$; $\mathbf{P}_1 \in \Re^n$
2. Feasibility Check
   - -produce volume predictions for all links from t=1,2,...H
   - -if infeasible, generate new **P**
   - -otherwise, goto 3.
3. Update $T_{i,j}(t)$ Values
   - -convert time-dependent volumes on each link into corresponding $T_{i,j}(t)$ values
4. Router (f)
   - -route optimally based on supplied $T_{i,j}(t)$ values
5. Optimality Check
   - -if $\mathbf{P} = f(\mathbf{P})$ then stop.
   - -otherwise, goto 2.

It is not obvious, however, that this process will necessarily terminate at an optimal point. In fact, more critically, it is not obvious whether after some number of iterations the process would converge at all. Unless at some pair of successive iterations, our decision vectors are exactly the same ($\mathbf{P}_i = \mathbf{P}_{i+1}$), then the process does not have the desired property of invariance and then no terminating solution can be found, optimal or otherwise.

Consider this crude algorithm in relation to our sample problem, using the $\mathbf{P}_1$ decision vector as the initial fixed point. The related travel times are then calculated (Table 2) and passed to the anticipatory routing algorithm. Note that no combination of travel times for the "long" route is better than the travel times for the "short" route, that is, $T_{12}(t)$

+ $T_{23}(t+T_{12}(t)) >= T_{13}(t)$ for all t = 1,2....6. It is clear, then, that the router will return instructions to route all vehicles along the "short" route, going directly from node 1 to node 3. The resultant decision vector, $\mathbf{P}_2$ = [1, 1, 1, 1, 1, 1], is used to calculate new $T_{ij}(t)$ values. However, as is demonstrated in Figs. 5-6, during the calculation of speeds on link 1-3 at the beginning of time period 2, we attempt to place vehicles #2 and #3 onto a link which already has three vehicles on it. The placement of the fourth vehicle, #3, onto the link generates a link speed of 4/3-4/3 = 0. Travel time on link 1-3 now becomes infinite, as the vehicles themselves are now "trapped" in positions on a link with s = 0. The travel times under decision vector $\mathbf{P}_2$ are given in Table 4:

TABLE 4. Travel times under $\mathbf{P}_2$ = [1, 1, 1, 1, 1, 1])

link	t=1	t=2	t=3	t=4	t=5	t=6	[time units]
1-2	1.5	.75	.75	.75	.75	.75	
1-3	3	∞	∞	∞	∞	∞	
2-3	.75	1	.75	.75	.75	.75	

It does not take a great deal of analytical perspicacity to see that if these travel times are then given back to the router for the next iteration, the result will be that all vehicles will be routed along the "long" route ($\mathbf{P}_3$ = [0, 0, 0, 0, 0, 0]). Our iterative process ends up rather quickly degenerating into a divergent alternating series of values which are not only sub-optimal, but worst-case. Obviously, our iterative method requires some significant improvements in order to have produce "good" solutions with properties like invariance, feasibility, and optimality. In section 3, the role of the link capacity or impedance function (like the 4/3 -v/3 Greenshields equation used in our example) is investigated in relation to its ability to accurately model observed traffic phenomena. Next, in section 4, there is a discussion of the structure of the iterative process itself and a review of some current journal articles in transportation research which also deal with the problem of divergence in other iterative assignment models.

Fig 5. Network status at the end of the 1st time period, $\mathbf{P} = [\ 1, 1, 1, 1, 1, 1\ ]$

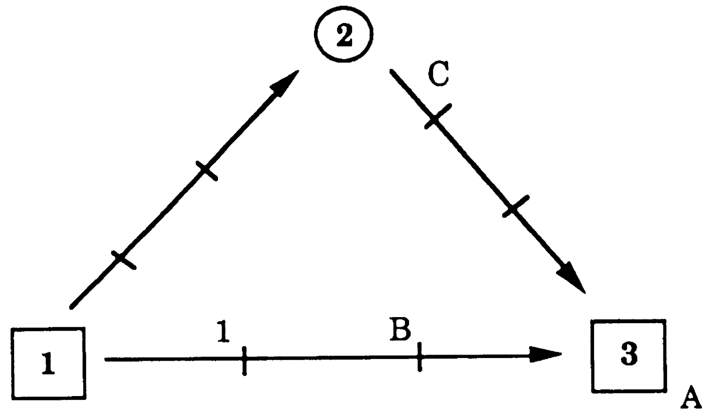
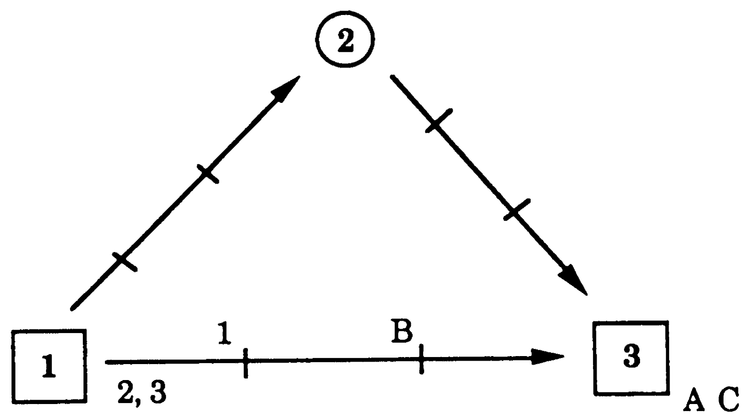


Fig 6. Network status at the end of the 2nd time period, $\mathbf{P} = [\ 1, 1, 1, 1, 1, 1\ ]$



- volume on link 1-3 is 4 vehicles
- speed is 4/3 - v/3 = 0
- travel time on this link is now infinite

3. THE ROLE OF LINK CAPACITY FUNCTIONS

The simple Greenshield's model used in our sample problem implies that there is some physical limit or capacity to the number of vehicles which can occupy space on a link. If one couches the discussion of impedance as a function of flow (vehicles passing a certain point on a link in a given time), then this physical capacity is called the "steady-state" capacity--the maximum number of vehicles which may be allowed to pass a certain point during a unit of time. The idea of a link having some sort of finite capacity appeals to our intuition because it is easy to conceptualize. At some point, we reason, so many vehicles can be loaded onto a link before travel speeds will become so reduced as to be indistinguishable from motionlessness. Therefore, in crude models like Greenshield's infinite travel times for links may be obtained, as reflected by the asymptote in a plot of such a function at the steady-state capacity. This, however, seems counter-intuitive because although some cars on the link may not be moving, there must be someone at the head of the queue who has no one in front of him (presumably) and may move off the link.

More sophisticated asymptotic models avoid this predicament by requiring actual flows to be restricted to less than the steady-state capacity. "Delay time" is modelled separately from "travel time" on the link--if a vehicle attempts to enter a link which is at capacity, it must wait until space has opened up in front of it. Although asymptotic models have the advantage that they are more realistic in relation to link capacity, they are more difficult to use in an iterative procedure because routings may now at worst produce infinite travel times or at best involve some complex bookkeeping method for delay and travel time.

Non-asymptotic models may instead by used, like the Bureau of Public Road (or BPR) curves often found in transportation textbooks. A typical BPR curve relates travel time to link flows in the following manner:

$$T = T_0 \, [\, 1 + a\, (Q/C)^b \,];$$ where T = travel time, Q = link flow, C = capacity, a, b = parameters

Queuing and stoppage on the link are assumed to be implicitly modeled in the function, and regardless of how many vehicles are on the link, we are guaranteed of finite (although perhaps very large) travel times. However, an optimal point generated using such a link capacity function would have to be carefully assessed since it may route vehicles in a manner which actually exceeds the measured physical capacity of a link.

An review of link capacity functions used in practice is provided by Branston (1976). A key point brought up in this paper is that regardless of whether the function is asymptotic or non-asymptotic in nature, it makes one critical assumption: flow and travel times on links are sampled over periods during which the effects of any time-dependent variations in the mean values are smoothed over, i.e., steady-state conditions. Quoting from Potts and Oliver (1972), p. 19: "The steady state conditions imply for traffic applications that we are not concerned with the microscopic and stochastic characteristics of a traffic stream...we ignore fluctuations over time." This is not surprising, since many link capacity functions were developed to closely approximate specific observed traffic flow data averaged over some relatively long period. However, in the context of dynamic route assignment, the larger our time units, the less accurately we may model resultant travel times--but we must weigh that gain in accuracy against the cost of increased computational requirements. This has already become an issue in the use of link capacity functions for equilibrium (non-dynamic) iterative algorithms (Taylor, 1984).

Also, the nature of the inputs to the models must be carefully considered. Is the flow on a link to be measured at the beginning of the link or the end? One may imagine that if an incident occurs somewhere in the middle of a link completely blocking vehicle movement, that some flow-measuring device at the end of the link would indicate that there was no traffic on the link whatsoever--which would only cause additional vehicles to be routed there and thus create a potentially disastrous situation. In addition, a simple function dependent only on data from a single link cannot model the interaction between links when traffic becomes so congested that vehicles are restricted from entering certain "overloaded" links ahead of them.

If simple functions are not able to accurately model all traffic phenomena, one may argue that they should be replaced by more sophisticated analytical techniques. Certainly, we could attempt to obtain travel time through convex analysis approaches similar to those outlined in Smith (1979) or extract travel times from some large scale mixed integer program related to Merchant and Nemhauser (1978), but these methods are so cumbersome and computationally complex that by the time values are produced for a single iteration, vehicles seeking route guidance would have long ago reached their destinations. Any effective real-time iterative process, I believe, must rely upon a fast and simple functional model, asymptotic or otherwise, to model flow/travel time relationships. No model will be perfect, but certain promising candidates may be altered to provide the requisite level of accuracy. For example, an extension of a Davidson's function in Taylor (1984) provides for the identification and elimination of infeasible routing strategies. Other models may be similarly extended....

4. INVARIANCE AND ITERATIVE STABILITY

Although the role of the link capacity function is significant, far more critical to the success of an iterative algorithm is the property of invariance. If the sequence of fixed point iterates cannot be shown to converge to a single optimal solution within a contraction mapping, then any successive approximation method is doomed to failure. Convergence to a fixed point in a complex network has not been proven theoretically, and is very difficult indeed to prove. It has been assumed by some researchers (Leonard, Tough and Baguley, 1978; Yagar, 1971; Alfa, 1987) that some fixed optimal point does exist and can be shown in a their examples. For our example problem, through a process of exhaustive enumeration, the decision vector $\mathbf{P}_* = [\,0, 1, 1, 0, 1, 1]$ can be shown to be a system-optimal solution with minimum average experienced travel time of 1.86 time units. If a dynamic router is provided with the travel times produced by $\mathbf{P}_*$, it will return $\mathbf{P}_*$, thus satisfying our property of invariance. Existence and uniqueness of a deterministic user equilibrium have been established only for a simple network with a single origin and destination (Smith, 1984; Daganzo, 1985).

So, for our example problem, a unique optimal fixed point does exist, but our iterative process did not find it. One reason for this lies in our method of assigning traffic to perceived shortest-routes without completely considering the congestion effects of assigning those vehicles. This "all or nothing" approach leads to instability precisely because those links with small flows and correspondingly small travel times are overloaded by a routing mechanism which does not model the immediate effects of its assignments.

One method which appears to produce relatively stable fixed points in a related context has been suggested by Alfa (1989). Alfa's task is to determine a dynamic equilibrium among commuters choosing departure times in the morning before leaving for work each day--not necessarily a shortest-path optimization problem, but similarly dependent on the ability to reconcile the "tail-chasing" inter-dependency of time-variant travel costs and routing decisions. His approach is to divide demands between O-D pairs into small equal groups and assume these groups travel together. The departure times for the first group (or first layer) from all the O-D pairs are then assigned based on some time-window in which each vehicle wishes to arrive at his destination. Volumes of traffic on links may be then calculated, and travel times updated so this information can be used to route the next group (layer) of vehicles. This process can then be iterated on a day-by-day basis until some equilibrium (or non-existence of equilibrium) is established for departure times.

While not directly involved with the establishment of optimal routing, the "layering" method is inherently more stable than a simple "all or nothing" assignment. My plans are to extend the rough outline of the successive approximation approach in the direction of a "layered" approach, together with improvements to the link capacity function, to see if enough such an approach can effectively find optimal routing vectors like $\mathbf{P}_*$. Once a suitable algorithm can be identified, the value of anticipatory routing techniques can be evaluated running on a sophisticated simulation platform like INTEGRATION.

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